On Single Crystal Strength Dependence on Size.

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Abstract.

Size effect is defined as strength dependence on size at a tensile test of a single crystal. The only geometric factor influences the effect is considered in this paper. Geometric factor includes two characters: specimen round cross-section and micron-sized diameter. The rest of the factors were not considered. Analysis of an equivalent model was fulfilled. It was revealed that the overcome of average specimen resistance leads to plastic changes, the elastic limit stress is inversely proportional to the specimen diameter. This conclusion is in good accordance with several experimental works relates to single crystal strength.

Key Words: single crystal, strength, geometric factor, elastic limit stress

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1. Introduction.

The phenomenon of plasticity strain beginning dependence on a specimen dimension has been chosen as a subject of the present analysis. The dependence was revealed at tensile tests of micron-sized specimens by a number of researches for different materials. Some of the dependence analysis relates to metal single crystals [1,2,3,4,5,6,7]. The term “strength” in most of them is used without a specific definition. In one case, the term means the plastic strain beginning stresses – proportionality limit, elastic limit - σ_e [5,8,9]. In another case – the maximal stress on the stress-strain diagram - ultimate stress - σ_u [1,2,5,8,10]. In the third – recalculated stress of indentation loading [4,8]. The approximate relation: $\sigma_u = \sigma_e + \text{Const.}$- is considered in the current work. It makes it possible to compare the results of different sources for the subject of “strength” dependence on dimensions. The dependence was perfectly demonstrated in a series of experiments with single-crystal whiskers, executed by J.J. Brenner in the middle of the 20-th century [1]. Similar results were published in [8,11,12], which concern nonmetal specimens (glass filaments, polyethylene fibers). Moreover, the same size dependence was obtained in compression tests of single-crystal micro specimens [8,13,14]. The authors of the signed works above give variable explanations of the phenomenon – “strength” dependence on specimen dimensions. Here not the whole plasticity process is considered, but only the relations preceding to it. It makes the subject of the present work become determined and suitable for analysis.

The only factor common to all signed above samples, the geometric one, was not taken into account. The two attributes that are common to all the samples are cylindrical shape (round cross-section) and micron range of diameters, by [3] diameters less than 10µm. Therefore, it's worth searching an explanation of the phenomenon in geometric relations.

2. The geometric relations analysis.

2.1 The plastic flow beginning model. The elastic component of strain.

First, the tension of micro specimens with a specific cylindrical shape (round cross-section) is considered.

The mechanism of the plastic strain start of single-crystal at tension was investigated in the previous author's work [15]. The mechanism was presented as an unstable surface atoms propagation inward the specimen volume. Elastic strain limit was defined as the beginning of abrupt increasing of the propagation acts' probability. Thus, an irreversible mass transfer occurs in a radial direction, transversal to the tension axis.

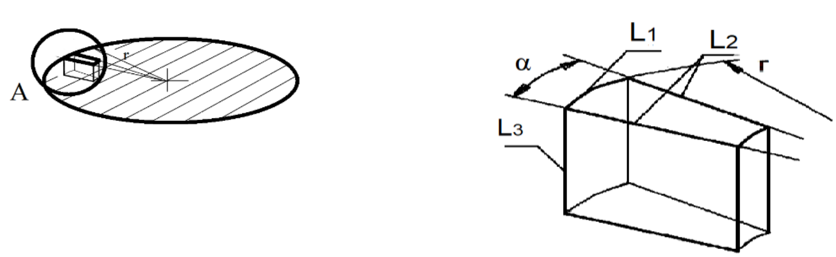
2.1.a Continual model of strain state.

The achievement of elastic limit in a continual model is characterized by the following values of base parameters:

- tensile stress – σ_e ;
- longitudinal strain - ε_e ;
- transverse strain – $\varepsilon_r = \eta \times \varepsilon_e$, where η – Poisson ratio;
- specimen diameter of micrometer range b = 2r , where r – radius.

The principal data for analysis is the following: specimen cylindrical shape (round cross-section), shape change resistance (solid body), and transverse direction of mass transfer. The last coincide with the direction of minimal energy losses.

The part of the specimen volume on the surface in cross-section is considered, Fig.1.



Detail A

a b

Fig. 1 a – Specimen cross-section; b – the volume part of single penetration act influence.

The part is defined as the most probable zone of influence of the single penetration act. The part is limited: by surface with the arc L_1; along the axis – a line L_3 of indefinite length; inward the volume – with two sections of radiuses L_2 from ends of arc L_1 and closing arc in section plane. Radial directions by the L_2 sides are defined from the axisymmetric relation. The adjacent parts are in the same relations of strain and loads. Side facets form a small angle α between themselves:

$$\propto = \frac{L_1}{r} . \qquad (1)$$

The equivalent virtual model of the stress state may be used here for the strain resistance appraisal. The model may be presented as the part, with applied to the surface outer virtual pressure p. This pressure causes the same transverse strain ε_r . As far as the loading occurs in the elastic range, the following is correct:

$$p = k \times \sigma_e \text{ , where k = const.} \qquad (2)$$

On side planes of segment (the part), tangential compressing stresses – σ_t , maximal at the surface – σ_{te} , act. In accordance with axial symmetry (neighbor segments are identical by sizes and shape), the last – σ_{te} makes sense of the elastic limit of material resistance. Any additional displacement or deflection of a surface causes the plastic flow. The integral segment equilibrium – from the projection of all external forces to symmetric axis:

$$p \cdot L_1 \cdot L_3 - 2\left(\int_0^{L_2} \sigma_t \cdot dl\right) \cdot L_3 \cdot \sin {}^{\alpha}/_{2} = 0 . \qquad (3)$$

In this case, it would not be correct to engage the Lame's solution of the axisymmetric problem to define resistance. Its usage may be physically incorrect without a strain compatibility condition. The function of stresses σ_t distribution along L_2 is not principal for the problem to be solved. Hyperbolic function Lame has a singularity at l = 0. Therefore, it is considered rightful to approximate with random decreasing power function:

$$\sigma_t(l) = -A \cdot l^n + B \quad : \text{ where } \quad n \geq 0 \qquad (4)$$

From border relations:

$$\sigma_t(l = 0) = \sigma_{te} \text{ ; B = } A \cdot {L_2}^n;$$

$$\sigma_t(l = L_2) = 0 \text{ ; } A = {}^{\sigma_{te}}/_{L_2^n} \text{ ; } B = \sigma_{te} \qquad (5)$$

$$\int_0^{L_2} \sigma_t \cdot dl = \sigma_{te} \cdot \int_0^{L_2} \left(-{}^{l^n}/_{L_2^n} + 1\right) dl =$$

$$\sigma_{te} \times [-\frac{l^{n+1}}{L_2(n+1)} + l] \uparrow_0^{L_2} = \sigma_{te} \times L_2 \times \left(\frac{n}{n+1}\right) . \qquad (6)$$

The principal feature from (6), that integral resistance is proportional to the maximal stress. After substitution (6) to (3):

$$p \times L_1 - 2\sigma_{te} \times L_2 \times \frac{n}{n+1} \times \frac{\propto}{2} = 0 \text{ ; with (1) :}$$

$$p \times L_1 - \sigma_{te} \times L_2 \times \frac{n}{n+1} \times \frac{L_1}{r} = 0 \text{ ; from last :}$$

$$p = \sigma_{te} \times \frac{L_2}{r} \times \frac{n}{n+1} \ . \qquad (7)$$

From (2) : $\sigma_e = \frac{p}{k} = \frac{\sigma_{te}}{k} \times \frac{L_2}{r} \times \frac{n}{n+1}$. (8)

The last formula's physical sense – the plastic flow at tension begins when the load exceeded the resistance to transverse mass transfer.

Parameters included in (8):

- σ_{te} – material resistance limit to transverse mass transfer;
- L_2 – length character, the depth of single penetration act influence (material parameter) ;
- r – radius, geometric parameter.

2.1.b The part influence zone in atomic structure scale.

The dependence σ_e of r by (8) is rightful in a rather limited range of parameters, but it seems suitable for size effect appraisal. In [15, 3.3] the minimal values of influence zone for the micro specimen of Fe single crystal was appraised. The arc L_1 by surface: $L_1 = 12.6 \times 2.86\dot{A} = 36.04 \times 10^{-10}$ m, where d = $2.86\dot{A}$ is the lattice parameter. With the specimen diameter 1μm, r = 0.5 μm, angle :

$$\alpha = {}^{36.04 \times 10^{-10}}\!/_{500 \times 10^{-9}} = 0.00721 rad \approx 0.42 grad \ .$$

3. Size effect at compression.

In a physical sense, the uniaxial compression test of a single-crystal is a "mirror reflection" of the tension process. Hence, the size effect may appear in this case also. The "mirror reflection" requires an inversed plastic flow process – exit to the surface of the unstable atoms from the inner volume. This state should be the subject of additional investigation.

4. Size effect at the indentation test.

Micron-sized specimens "Strength" at the test with cylindric indenter appraised by the size of the imprint. The size effect (dependence on specimen diameter) revealed here is in good correspondence with the accepted calculation method of contact stresses, which depend on the radiuses of the indenter and specimen [8,16]. Hence, the geometric factor, in this case, is also significant. It's worth noting that a comparison of two "strength" appraisal methods, indentation and tension, may be problematic. Material resistance limits to contact load, and tension differs famously.

5. Conclusion.

The dependence (8) comparison with diagrams of experiments data by [1,2,5,8,9] shows that (8) is in good agreement with the data dispersion. However, [12]: "ultimate strength of the polyethylene fibers was found to be inversely proportional to the square root of its diameter." But, the last might also be accounted as a geometric factor significant influence.

Statements and Declarations

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